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The shape of gravity

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Abstract: In a non-trivial background geometry with extra dimensions, gravitational effects will depend on the shape of the Kaluza-Klein excitations of the graviton. We investigate a consistent scenario of this type with two positive tension three-branes separated in a five-dimensional anti-de Sitter geometry. The graviton is localized on the "Planck" brane, while a gapless continuum of additional gravity eigenmodes probe the *infinitely* large fifth dimension. Despite the background five-dimensional geometry, an observer confined to either brane sees gravity as essentially four-dimensional up to a position-dependent strong coupling scale, no matter where the brane is located. We apply this scenario to generate the TeV scale as a hierarchically suppressed mass scale. Arbitrarily light gravitational modes appear in this scenario, but with suppressed couplings. Real emission of these modes is observable at future colliders; the effects are similar to those produced by *six* large toroidal dimensions.

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Extra dimensions provide an alternative route to addressing the hierarchy problem. This is because the Planck scale, describing the strength of the graviton coupling at low energies, is a derived scale. In a simple factorizable geometry, the Planck scale of a four-dimensional world is related to that of a higher-dimensional world simply by a volume factor. The large Planck scale indicates weak graviton coupling which is in turn a consequence of the large volume over which the graviton can propagate [1]. In this scenario, a large hierarchy only arises in the presence of a large volume for the compactified dimensions, which is very difficult to justify. A more compelling alternative has been suggested in ref. [2]. The idea of this paper was that the weak graviton coupling arises because of an interesting shape of the graviton wave function in the extra dimensions. The graviton is localized away from the $(3+1)$-dimensional world on which the standard model resides. The large value of the Planck scale arises because of the small amplitude for the graviton to coincide with our "brane".

In ref. [3], it was shown that the geometry of a single brane with cosmological energy densities tuned to guarantee Poincaré invariance takes the form:

$$ds^2 = e^{-2k|y|}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + dy^2\,, \tag{1}$$

where μ, ν parameterize the four-dimensional coordinates of our world, and y is the coordinate of a fifth dimension. The remarkable aspect of the above geometry is that it gives rise to a localized graviton field. Mechanisms for confining matter and gauge fields to a smaller-dimensional subspace were already known. The new feature here is that the background geometry gives rise to a single gravitational bound state. This mode plays the role of the graviton of a four-dimensional world, and is responsible for reproducing four-dimensional gravity. In ref. [3], the Kaluza-Klein (KK) spectrum reflecting the large extra dimension was derived and it was argued that the additional *continuum gapless* spectrum of states gives rise to very suppressed corrections to conventional four-dimensional gravity, suppressed by $(\text{energy}/M_{Pl})^2$.

However, from the perspective of generating the mass hierarchy between the Planck and weak scales, the important aspect of this geometry is the correspondence between location in the fifth dimension and the overall mass scale. This can be understood by the fact that the warp factor is a conformal factor so far as a four-dimensional world located at a fixed y location is concerned. Mass factors are rescaled by this factor, so that a natural scale for mass parameters might be $M_{Pl} = 10^{19}\,\text{GeV}$ on a brane at the origin, but is $M_{Pl} \exp -k|y|$ for physics confined to a location y. This *exponential* could be the source of the hierarchy between the electroweak scale of order TeV and the Planck scale which is approximately 10^{15} times bigger. Notice that the generation of this hierarchy only requires an exponential of order 30.

In ref. [2], this observation was exploited by introducing an orbifold geometry, and located a positive energy brane at one point and a negative energy brane at the

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second orbifold point. If the standard model is located on this second, negative energy brane, the amplitude of the graviton is exponentially suppressed and a hierarchy is generated.

The potential disadvantage of this setup is the necessity for the negative energy object and the orbifold geometry. Although not ruled out, it is desirable to have an alternative setup involving only positive energy objects. The advantages of such a setup are as follows. First, there are positive energy objects, namely D-branes and NS-branes, that are well understood and on which gauge fields and matter fields can be localized so that the standard model fields can be placed there. Finally, there is the aesthetic advantage of allowing for an infinite-dimensional space in which mass scales are associated with definite locations in the space, a point further emphasized by [4]. If one permits all possible mass scales (all possible distances in the fifth dimension), one presumably has a better chance of addressing difficult cosmological issues such as the cosmological constant problem and black-hole physics. One also has a better chance for exploiting holographic ideas by exploiting the correspondence between location in the fifth dimension and mass scale.

In this paper, we demonstrate that one can address the hierarchy problem with only positive energy objects by combining the two observations of ref. [2, 3], namely (1) it is consistent to live with an infinite fifth dimension, and (2) one can generate a hierarchy by living far from the brane on which gravity is localized. This was implicit also in ref. [4], where the connection between distance in the fifth dimension and overall mass scale was made explicit in an AdS geometry derived from D-3 branes (so that the Maldacena conjecture [5] could be exploited), and so that the TeV scale corresponded to a fixed coordinate y_0. Similar ideas were also discussed in [6].

The crucial question is whether an observer on this "TeV brane" sees a consistent theory of gravity. In ref. [3], it was only shown that one sees a theory of gravity that is very close to a four-dimensional gravitational theory if one lives on the brane on which the graviton is localized. In this paper, we argue that even for an observer quite far from that brane, one obtains an acceptable gravitational theory, essentially indistinguishable from a four-dimensional world!

The picture that emerges is remarkably beautiful. The graviton is localized on a brane that we call the Planck brane from now on. We live on a brane separated from the Planck brane by roughly 30 Planck lengths in the fifth dimension. On this brane, mass scales are exponentially suppressed, yielding a natural generation of the weak scale. Furthermore, the maximal scale we can probe on our brane is this same TeV scale, since all string modes become strongly interacting at this scale. The location of the brane, which we denote by y_0, was determined to give the correct ratio of the weak scale to the Planck scale. We call the brane at this location the TeV brane.

The potentially dangerous aspect of this setup is the multiplicity of the arbitrarily light Kaluza-Klein modes. In ref. [2], it was argued that the KK modes were extremely strongly coupled (with TeV coupling suppression rather than M_{Pl}). In ref. [3], it was shown that one signal of the infinite extra dimension is a gapless continuum of Kaluza-Klein modes. Clearly, if these modes were all so strongly coupled, the theory would be disastrous, since gravitational and particle physics tests would be badly violated.

What we show in this paper is that the situation is far more clever. Production of modes less than the TeV scale are suppressed. Furthermore, modes less than 10^{-4} eV (which happens to correspond to the length scales on which gravity has been directly probed), still couple with Planck scale suppression. Thus the theory interpolates between a four-dimensional and five-dimensional world (reminiscent of a holographic interpretation). The observer on the brane at the TeV scale sees the modes below a TeV in energy as weakly coupled. Modes higher in mass than a TeV are much more strongly coupled, and would in principle reproduce the expected five-dimensional result. However, they are impossible to access! Generalizing to an arbitrary location, one never recognizes the higher-dimensional geometry. Independent of location, the world appears lower-dimensional at low energies.

We now elaborate on this observation. The results follow readily from papers [2, 3]. Our setup is a "Planck brane" (or set of branes) on which the graviton zero mode is confined, exponentially falling off in the direction y. The new feature is a single brane (or multiple branes) located a distance y_0 from this brane, where $e^{-ky_o} = \mathrm{TeV}/M_{Pl}$, where k is related to the cosmological constant on the brane and determines the exponential falloff of the graviton, as in ref. [2, 3]. The new brane can be regarded as a probe of the geometry determined by the Planck brane, either by assuming that the Planck brane has much larger tension, or consists of a large set of branes. It is readily seen that inclusion of a small brane tension does not substantially affect the result. We also remark that we do not address the question of determining the location y_0 here, though mechanisms that stabilize the orbifold geometry (such as in ref. [7]) should also apply.

It is clear that the zero mode generates consistent gravity. If we take the coordinate $y = 0$ to be the location of the Planck brane, one can readily derive:

$$M_{Pl}^2 = 2\int_0^\infty dy\, e^{-2ky}\, M^3 = \frac{M^3}{k}\,, \tag{2}$$

so that with M and k taken of order $M_{Pl} = 10^{19}$ GeV, the zero mode is coupled correctly to generate four-dimensional gravity.

It is therefore the contribution of the additional KK modes that is our focus. Everything follows from the detailed form of these modes, derived in [3]. The graviton zero mode (properly normalized) is

$$\hat{\Psi}_0(z) = \frac{1}{k(|z| + 1/k)^{3/2}}\,, \tag{3}$$

where the coordinate z is related to y by the expression

$$z = \frac{\operatorname{sgn}(y)}{k}\left(e^{k|y|} - 1\right). \tag{4}$$

Note that at the TeV brane $z = z_0 \sim 1\,\text{TeV}^{-1}$.

The continuum KK modes are given by:

$$\hat{\psi}_m \sim N_m \left(|z| + \frac{1}{k}\right)^{1/2} \left[Y_2\left(m\left(|z| + \frac{1}{k}\right)\right) + \frac{4k^2}{\pi m^2} J_2\left(m\left(|z| + \frac{1}{k}\right)\right)\right], \tag{5}$$

where m is the mass of the mode, Y_2 and J_2 are Bessel functions, and N_m is a normalization factor.

For large mz, these modes asymptote to continuum plane wave behavior. This can be seen from the asymptotic form for the Bessel functions:

$$\sqrt{z}\, J_2(mz) \sim \sqrt{\frac{2}{\pi m}} \cos\left(mz - \frac{5}{4}\pi\right), \qquad \sqrt{z}\, Y_2(mz) \sim \sqrt{\frac{2}{\pi m}} \sin\left(mz - \frac{5}{4}\pi\right). \tag{6}$$

The normalization constants N_m are determined by this plane wave behavior [3]:

$$N_m \sim \frac{\pi m^{5/2}}{4k^2}. \tag{7}$$

We are adopting here a delta-function normalization such that physical quantities will always involve an integration over m for which the proper measure is just dm. None of our calculations will involve any dependence on the $y \to \infty$ regulator scheme (i.e. the "regulator brane" of [3] or the alternative proposed in [4]).

It is edifying to consider what these modes tell us in a couple of limiting situations. First, let us remind the reader of what happens if you live on the Planck brane ($z = 0$). The exact effect depends on the particular gravitational process under consideration; let us first consider the corrections to Newton's law from the KK modes. One finds a potential between two masses m_1 and m_2:

$$V = G_N \frac{m_1 m_2}{r} + \int_0^\infty \frac{dm}{k}\, G_N \frac{m}{k} \frac{m_1 m_2\, e^{-mr}}{r} = G_N \frac{m_1 m_2}{r}\left(1 + \frac{1}{k^2 r^2}\right). \tag{8}$$

The KK contribution is suppressed at large distances over and above that expected from having one additional dimension, because of the amplitude suppression near the brane. This is due to the barrier of the analog quantum mechanical problem used to find the KK modes.

Now let us consider the opposite extreme. Suppose we were at high energies and *suppose* it were appropriate to use the plane wave form of the modes. At a given location y_0, what would be the corrections to Newton's law? They would be

$$V \sim G_N \frac{m_1 m_2}{r} + \int_0^\infty \frac{dm}{k}\, G_N \frac{m_1 m_2\, e^{-mr}}{r} e^{3ky_0} \sim G_N \frac{m_1 m_2}{r}\left(1 + \frac{e^{3ky_0}}{kr}\right). \tag{9}$$

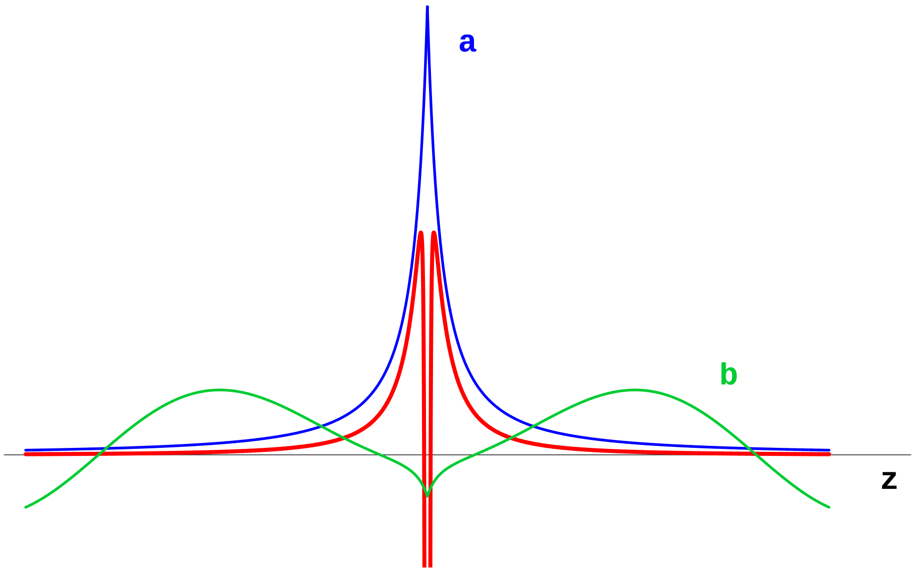

Figure 1: The graviton Kaluza-Klein modes are determined from an analog one-dimensional quantum mechanics problem. The "volcano" potential is shown in red. Also shown are (a) the localized zero mode, and (b) an example of the gapless continuum spectrum of additional KK modes.

It is useful to write this answer in terms of mass scales and compare to a flat five-dimensional space. If y_0 is chosen to address the TeV hierarchy, one finds the correction factor $(M_{Pl}/\mathrm{TeV})^3/kr$. Taking $k \sim M_{Pl}$, one derives $M_{Pl}^2/\mathrm{TeV}^3 r$. With a cross product background metric, one would derive the TeV scale by choosing r_c as $M^3 r_c = M_{Pl}^2$ where M is of order a TeV and the mass of the KK modes would be starting at $1/r_c$. The corrections to gravity would be those of the number of modes of energy less than $1/r$, which would be M_{Pl}^2/rM^3. This precisely agrees with the contribution one would have in the warped background if one saw the full continuum contribution. This would of course be ruled out by current experiments as it is far too strong a correction to gravity.

However, the calculation in the AdS background based on the continuum form of the KK modes is not appropriate. It helps to examine the detailed form of the KK modes. Recall that they were derived in a background AdS space in which there was a four-dimensional flat brane with localized energy density. One derives the KK modes by assuming they factorize into momentum eigenstates with mass m, where m is determined by solving an analog quantum mechanics problem describing the shape of the KK mode in the fifth dimension. The analog potential for these modes was dubbed the "volcano" potential because there was a delta-function at the origin, a barrier, and then a smooth fall-off to zero. The zero mode is the single bound state. All other modes are suppressed at the origin (as the first calculation of corrections to Newton's law showed) and then turn into continuum plane wave modes in the large y region, far from the brane.

At a given location y_0, modes which are sufficiently light are suppressed relative to their continuum form, while modes which have already assumed their asymptotic form are unsuppressed. We can quantify this statement by examining the explicit expression for the modes eq. (5). The asymptotic forms of the Bessel functions, and thus the onset of continuum behavior, requires mz_0 much greater than 1, which is only true for modes of mass greater than a TeV. This is an important result. It says that modes at all energies below the strongly interacting regime are more suppressed than a continuum KK mode. This result could have been anticipated from ref. [2], where it was shown that quantization was in units of approximately TeV. Modes do not appear to have their continuum form until they are at least this massive.

The suppression of the lighter modes is addressed by looking at the asymptotic form of the Bessel functions for small mz:

$$\sqrt{z}\,J_2(mz) \sim \frac{m^2}{8}\,z^{5/2}\,, \qquad \sqrt{z}\,Y_2(mz) \sim -\frac{4}{\pi m^2 z^{3/2}} - \frac{z^{1/2}}{\pi}\,. \tag{10}$$

We see that Y_2 tracks the zero mode, whereas J_2 rises sharply with respect to the zero mode. So long as Y_2 dominates, the contribution from the KK modes is as suppressed relative to that of the zero mode as if we were probing gravity on the Planck brane; e.g. the corrections to Newton's law are given by eq. (8). We find that Y_2 dominates so long as we are exploring modes with mass less than $1/(kz_0^2)$, which is approximately $10^{-4}\,\mathrm{eV}$. All gravitational experiments to date see the corrections to gravity to be as small as if we were living on the Planck brane!

Modes with masses in the region intermediate between $10^{-4}\,\mathrm{eV}$ and $1\,\mathrm{TeV}$ are controlled by the small mz behavior of the dominant J_2 term. If these modes had already reached their continuum form at $z = z_0$, the cross section for real emission of these modes would be proportional to $E/(\mathrm{TeV})^3$, where E denotes the relevant physical energy scale. This agrees with [8] for $n = 1$ extra dimensions, and leads to astrophysical and collider effects which are clearly excluded by observations. Using the actual form of these modes at z_0, we find instead that the real emission cross section is proportional to

$$\sigma \sim \frac{E^6}{(\mathrm{TeV})^8}\,. \tag{11}$$

So in fact the leading order energy dependence of these modes agrees with the large torus compactifications of [1] for the case of $n = 6$ extra dimensions! Because these effects are much softer in the infrared, they turn out to be easily compatible with all existing observations [8].

In fact, a stronger result readily follows. If matter is localized to any four-dimensional flat brane between the Planck and TeV branes, the force between the matter will look four-dimensional for energies less than a TeV. This means that one could imagine doing physics in the bulk, analogous to what one might have tried in the orbifold case, to explain features of our observable world.

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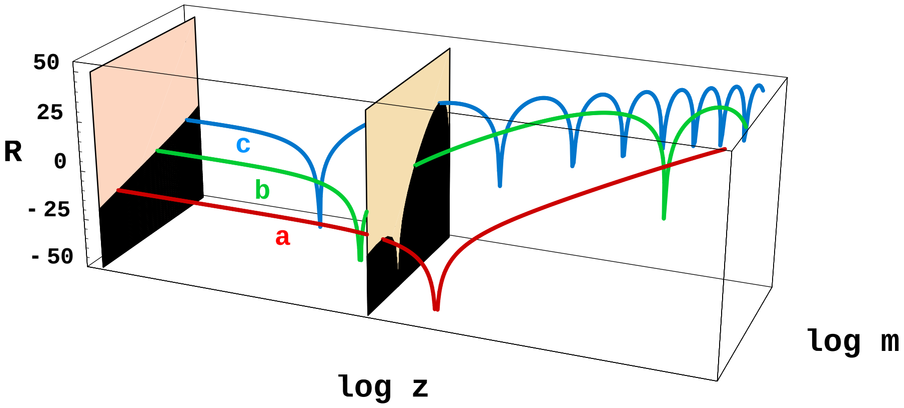

Figure 2: Schematic view of the Kaluza-Klein gravity modes. Plotted is $R = \log(|\psi_m|/|\psi_0|)$, the logarithm of the amplitude of the mode $\psi_m(z)$ relative to the amplitude of the graviton zero mode $\psi_0(z)$. The horizontal axes represent the coordinate z of the fifth dimension and the mass m of the gravity modes, both on logarithmic scales. The shaded vertical slices show the behavior of the modes at the Planck and TeV branes. Mode (a) is representative of the very light modes with $m < 10^{-4}\,\mathrm{eV}$; (b) is representative of modes with $10^{-4}\,\mathrm{eV} < m \ll 1\,\mathrm{TeV}$, and (c) shows the oscillating form of the heavy $m > 1\,\mathrm{TeV}$ modes.

What emerges is a very compelling picture. The world is five-dimensional: the coordinate y extends to infinity. However, for any observer localized to a given location y_0, the modes of mass greater than $M_{Pl}\ \exp -k|y_0|$ are strongly coupled. The amplitude of lighter modes on the y_0 brane is suppressed. Those of mass less than $1/kz_0^2$ ($10^{-4}\,\mathrm{eV}$ in our case) are coupled by $1/M_{Pl}$ with further amplitude suppression. Heavier modes are power law suppressed over what would be expected had the metric been flat. So the observer confined to the brane sees gravity as essentially four-dimensional, no matter where the brane is located! We can live with an infinite extra dimension and simply not know it.

The scenario presented here will be tested at future collider experiments. To leading order in (E/TeV), real emission effects will mimic those of $n = 6$ extra dimensions in the scenario of [1]. Because of the strong power suppression, it is important to be able to probe energy scales close to $(1/z_0)$. There may also be detectable effects from virtual exchanges of KK modes. However such effects are difficult to compute since they are dominated by heavy modes near the TeV cutoff; a string theory calculation is probably required to get a reliable estimate.

This is a very tantalizing scenario. It clearly ties in well with the holographic picture advocated in [4]. Again, with the infinite dimension, one expects the gravitational theory to correspond to a gauge theory cut-off in the ultraviolet. Within this theory, there is a correspondence between location y and mass scale determined by the shape of the zero mode. The additional contribution of this paper is to demonstrate that the Kaluza-Klein excitations do not disturb this picture. They give small corrections to the theory of gravity, so long as one is at sufficiently low energy. This new venue should provide new avenues for addressing important problems in cosmology and gravity.

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References

[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315];
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *New dimensions at a millimeter to a fermi and superstrings at a TeV*, *Phys. Lett.* **B 436** (1998) 257 [hep-ph/9804398].

[2] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[3] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[4] H. Verlinde, *Holography and compactification*, hep-th/9906182.

[5] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200];
O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rep.* **323** (2000) 183 [hep-th/9905111].

[6] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and N. Kaloper, *Infinitely large new dimensions*, *Phys. Rev. Lett.* **84** (2000) 586 [hep-th/9907209].

[7] W.D. Goldberger and M.B. Wise, *Modulus stabilization with bulk fields*, *Phys. Rev. Lett.* **83** (1999) 4922 [hep-ph/9907447].

[8] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phenomenology, astrophysics and cosmology of theories with sub-millimeter dimensions and TeV scale quantum gravity*, *Phys. Rev.* **D 59** (1999) 086004 [hep-ph/9807344].